# ON A CANONICAL FORMULATION OF STRING THEORY IN MASSIVE BACKGROUND FIELDS

I.L. BUCHBINDER, V.D. PERSHIN, G.B. TODER
*Department of Theoretical Physics, Tomsk State Pedagogical University, Tomsk 634041, Russia*

We propose a method of constructing a gauge invariant canonical formulation for non-gauge classical theory which depends on a set of parameters. Requirement of closure for algebra of operators generating quantum gauge transformations leads to restrictions on parameters of the theory. This approach is then applied to bosonic string theory coupled to massive background fields. It is shown that within the proposed canonical formulation the correct linear equations of motion for background fields arise.

The BFV method [1] is the most general realization of canonical quantization procedure providing a natural and consistent approach to construction of quantum models in theoretical physics. In this paper we discuss one of its yet unexplored aspects arising from bosonic string theory coupled to background fields [2].

A crucial point of string models is requirement of conformal invariance at the quantum level. It leads to restrictions on spacetime dimension in the case of free string theory and to effective equations of motion for massless background fields in the case of string theories coupled to background [2]. According to the prescription generally accepted in functional approaches to string theory [3] dynamical variables should be treated in different ways. Namely, functional integration is carried out only over string coordinates while components of two-dimensional metric are considered as external fields. This approach can also be applied to string theory interacting with massive background fields which is not classically conformal invariant [4]. As was shown in [5] it gives rise to effective equations of motion for massive background fields. Unfortunately, in the case of closed string theory covariant approaches did not reproduce the full set of correct linear equations of motion for massive background fields [5,6] and so there exists a problem of deriving correct massive background fields equations.

Moreover, from general point of view the requirement of quantum Weyl invariance of string theory with massive background fields means that a non-gauge classical theory depending on a set of parameters is used for constructing of a quantum theory that is gauge invariant under some special values of the parameters. As we consider canonical approach to be the only completely consistent method for constructing quantum theories so the general problem arising from string theories is how to describe in terms of canonical quantization

construction of gauge invariant quantum theory starting with a classical theory without this invariance.

Due to the general BFV method one should construct hamiltonian formulation of classical theory and define fermionic functional $\Omega$ generating algebra of gauge transformations and bosonic functional $H$ containing information of theory dynamics. Quantum theory is consistent provided that the operator $\hat{\Omega}$ is nilpotent and conserved in time. The corresponding analysis for bosonic string coupled to massless background fields was carried out in the paper [7]. In the case of string theory interacting with massive background fields classical gauge symmetries are absent and it is impossible to construct classical gauge functional $\Omega$. In this paper we propose a prescription allowing for some models to construct quantum operator $\hat{\Omega}$ starting with a classical theory without first class constraints. Then we apply it to the theory of closed bosonic string coupled to masssive background fields.

Consider a system described by a hamiltonian

$$H = H_0(a) + \lambda^\alpha T_\alpha(a) \tag{1}$$

where $H_0(a) = H_0(q,p,a)$, $T_\alpha(a) = T_\alpha(q,p,a)$ and $q$, $p$ are canonically conjugated dynamical variables; $a = a_i$ and $\lambda^\alpha$ are external parameters of the theory. We suppose that $T_\alpha$ are some functions of the form $T_\alpha = T_\alpha^{(0)} + T_\alpha^{(1)}$ and closed algebra in terms of Poisson brackets is formed by $T_\alpha^{(0)}$, not by $T_\alpha$:

$$\{T_\alpha^{(0)}(a), T_\beta^{(0)}(a)\} = T_\gamma^{(0)}(a)U_{\alpha\beta}^\gamma(a), \quad \{H_0(a), T_\alpha^{(0)}(a)\} = T_\gamma^{(0)}(a)V_\alpha^\gamma(a) \tag{2}$$

Such a situation may occur, for example, if $T_\alpha^{(0)}$ correspond to a free gauge invariant theory and $T_\alpha^{(1)}$ describe a perturbation spoiling gauge invariance. At the quantum level both the algebras of $T_\alpha^{(0)}(a)$ and $T_\alpha(a)$ are not closed in general case.

We define quantum operators $\Omega$ and $H$ as follows:

$$\Omega = c^\alpha T_\alpha(a) - \frac{1}{2}U_{\alpha\beta}^\gamma(a) : \mathcal{P}_\gamma c^\alpha c^\beta :, \quad H = H_0(a) + V_\alpha^\gamma(a) : \mathcal{P}_\gamma c^\alpha : \tag{3}$$

where $:\ \ :$ stands for some ordering of ghost fields. In general case $\Omega^2 \neq 0$ and $d\Omega/dt \neq 0$. However, if there exist some specific values of parameters $a$ that make the operator $\Omega$ to be nilpotent and conserved then the corresponding quantum theory is gauge invariant. Thus it is possible to construct a quantum theory with given gauge invariance that is absent at the classical level.

As an example where the described procedure really works we consider closed bosonic string theory coupled with background fields of tachyon and of

the first massive level in linear approximation.[a] The theory is described by the classical action

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma\,\sqrt{-g}\big\{\frac{1}{2}g^{ab}\partial_a x^\mu \partial_b x^\nu \eta_{\mu\nu} + Q(x)$$
$$+\, g^{ab}g^{cd}\partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda \partial_d x^\kappa F^1_{\mu\nu\lambda\kappa}(x) + g^{ab}\varepsilon^{cd}\partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda \partial_d x^\kappa F^2_{\mu\nu\lambda\kappa}(x)$$
$$+\, \alpha' R\, g^{ab}\partial_a x^\mu \partial_b x^\nu W^1_{\mu\nu}(x) + \alpha' R\,\varepsilon^{ab}\partial_a x^\mu \partial_b x^\nu W^2_{\mu\nu}(x) + \alpha'^2 R\,R\,C(x)\big\}, \quad (4)$$

$\sigma^a = (\tau, \sigma)$ are coordinates on string world sheet, $R$ is scalar curvature of $g^{ab}$, $\eta_{\mu\nu}$ is Minkowski metric of $D-$dimensional spacetime, $Q$ is tachyonic field and $F$, $W$, $C$ are background fields of the first massive level. As was shown in [5] all other possible terms with four two-dimensional derivatives in classical action are not essential and string interacts with background fields of the first massive level only by means of the terms presented in (4).

Components of two-dimensional metric $g_{ab}$ should be considered as external fields, otherwise the classical equations of motion $\delta S/\delta g_{ab} = 0$ would be fulfilled only for vanishing background fields. Such a treatment corresponds to covariant methods where functional integral is calculated only over $x^\mu$ variables. After the standard parametrization of the metric

$$g_{ab} \;=\; e^\gamma \begin{pmatrix} \lambda_1^2 - \lambda_0^2 & \lambda_1 \\ \lambda_1 & 1 \end{pmatrix} \quad (5)$$

the hamiltonian in linear approximation in background fields takes the form $H = \int d\sigma\,(\lambda_0 T_0 + \lambda_1 T_1)$ where $T_0 = T_0^{(0)} + T_0^{(1)}$, $T_1 = T_1^{(0)}$ and $T^{(0)}$ represent constraints of free string theory forming closed algebra in terms of Poisson brackets. In free string theory conditions $T^{(0)} = 0$ result from conservation of canonical momenta conjugated to $\lambda$. According to our prescription in string theory with massive background fields $\lambda$ can not be considered as dynamical variables, there are no corresponding momenta and conditions of their conservation do not appear.

The role of parameters $a$ is played by background fields and conformal factor $\gamma$ and the theory is of the type (1) with $H_0 = 0$. Direct calculations up to terms linear in bakground fields show that the operator $\Omega$ defined according to (3) is nilpotent and conserved in this theory under the following conditions:

$$D = 26, \quad \gamma = const, \quad (\partial^2 + 4/\alpha')Q = 0, \quad (\partial^2 - 4/\alpha')F_{\mu\nu,\lambda\kappa} = 0,$$
$$\partial^\mu F_{\mu\nu,\lambda\kappa} = 0, \quad \partial^\lambda F_{\mu\nu,\lambda\kappa} = 0, \quad F^\mu{}_{\mu,\lambda\kappa} = 0, \quad F_{\mu\nu,}{}^\lambda{}_\lambda = 0. \quad (6)$$

[a] An adequate treatment of non-linear (interaction) terms is known to demand non-perturbative methods [8].

The condition $\gamma = const$ means that string world sheet should be flat $R = 0$ and so the background fields $W$ and $C$ disappear from the classical action (4). The eqs.(6) show that first massive level is described by a tensor of fourth rank which is symmetric and traceless in two pairs of indices and transversal in all indices. This exactly corresponds to the closed string spectrum and so our approach gives the full set of correct linear equations for massive background fields.

The described example demonstrates a possibility to construct canonical formulation of quantum theory invariant under gauge transformations that are absent at the classical level. The proposed method opens up a possibility for deriving interacting effective equations of motion for massive and massless background fields within the framework of canonical formulation of string models and provides a justification of covariant functional approach to string theory.

The authors are grateful to E.S. Fradkin, P.M. Lavrov, R. Marnelius, B. Ovrut and I.V. Tyutin for discussions of some aspects of the paper. The work was supported by International Scientific Foundation, grant No RI 1300, and Russian Foundation for Fundamental Research, project No 96-02-16017.

1. E.S.Fradkin, G.A.Vilkovisky, *Phys. Lett.* **55B**, 224, (1975); I.A.Batalin, G.A.Vilkovisky, *Phys. Lett.* **69B**, 409, (1977); I.A.Batalin, E.S.Fradkin, *Ann. Inst. H. Poincare*, **49**, 145, (1988).
2. E.S.Fradkin, A.A.Tseytlin, *Phys. Lett.* **158B**, 316, (1985), *Nucl. Phys.* **261B**, 1, (1985); C.G.Callan, O.Friedan, E.Martinec, M.J.Perry, *Nucl. Phys.* **262B**, 593, (1985).
3. A.M.Polyakov, *Phys. Lett.* **B103**, 207, (1981).
4. J.M.F. Labastida, M.A.H. Vozmediano, *Nucl. Phys.* **312B**, 308 (1989); J.-C. Lee, B.A. Ovrut, *Nucl. Phys.* **336B**, 222 (1990).
5. I.L.Buchbinder, E.S.Fradkin, S.L.Lyakhovich, V.D.Pershin, *Phys. Lett.* **304B**, 239, (1993); I.L.Buchbinder, V.A.Krykhtin, V.D.Pershin, *Phys. Lett.* **348B**, 63, (1995).
6. V. Ellwanger, J. Fuchs, *Nucl. Phys.* **312B**, 95 (1989); J. Hughes, J. Liu, J. Polchinski, *Nucl. Phys.* **316B**, 15 (1989).
7. I.L.Buchbinder, E.S.Fradkin, S.L.Lyakhovich, V.D.Pershin, *Int. J. Mod. Phys.* **6A**, 1211, (1991). I.L.Buchbinder, B.R.Mistchuk, V.D.Pershin, *Phys. Lett.* **353B**, 257, (1995).
8. S.R.Das, B.Sathiapalan, *Phys. Rev. Lett.* **56**, 2654, (1986); C.Itoi, Y.Watabiki, *Phys. Lett.* **198B**, 486, (1987); A.A.Tseytlin, *Phys. Lett.* **264B**, 311, (1991).